# Using the TI-89 to find Riemann sums

If function $f$ is continuous on interval $[a, b]$, $\int_a^b f(x)\,dx$ exists and can be approximated using either of the Riemann sums

$$\sum_{i=1}^{n} f(x_i)\Delta x \qquad \text{or} \qquad \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

where $\Delta x = \frac{(b-a)}{n}$ and $n$ is the number of subintervals chosen. The first of these Riemann sums evaluates function $f$ at the right endpoint of each subinterval; the second evaluates at the left endpoint of each subinterval.

To evaluate $\sum_{i=1}^{n} f(x_i)$ using the TI-89, go to **F3 Calc** and select **4: ∑ ( sum**

The command line should then be completed in the following form:

$$\sum(\boldsymbol{f(x_i)},\ \mathbf{i},\ \mathbf{1},\ \mathbf{n})$$

The actual Riemann sum is then determined by multiplying this **ans** by $\Delta x$ (or incorporating this multiplication into the same command line.)

[**Warning:** Be sure to set the **MODE** as **APPROXIMATE** in this case. Otherwise, the calculator will spend an inordinate amount of time attempting to express each term of the summation in exact symbolic form.]

**Example:** To approximate $\int_2^4 \sqrt{1 + x^3}\,dx$ using Riemann sums with $n = 100$ subintervals, note first that $\Delta x = \frac{b-a}{n} = \frac{2}{100} = .02$. Also $x_i = 2 + i\Delta x = 2 + \frac{i}{50}$. So the right endpoint approximation will be found by entering

**∑(√(2 + (1 + i/50)^3), i, 1, 100) × .02**

which gives 10.7421. This is an overestimate, since the function is increasing on the given interval. To find the left endpoint approximation, replace $i$ by $(i - 1)$ in the square root part of the command. Alternately, just adjust the right-hand sum by subtracting the term with $x = 4$ and adding in a term with $x = 2$. The result will be 10.7411, an underestimate. Similarly, using $n = 1000$ shows that the integral lies between values 10.7365 and 10.7466, and using $n = 10000$ shows the value to lie between 10.7411 and 10.7421. (The TI-89 has a built-in approximate integration technique which can be accessed from **F3 Calc** and choosing option **2:** $\int$ **integrate**, which gives 10.7416 in this case.)

**Exercise:** Find upper and lower estimates for the value of $\int_0^3 \frac{1}{1 + x^7}\,dx$ using Riemann sums with $n = 1000$.

[**Answer:** The lower estimate (from right-hand endpoints) is 1.03265; the upper estimate (from left-hand endpoints) is 1.03565.]